Institute For
New Economic Thinking

The multiplex and Open Structure of the Italian Interbank Market

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  - Are layers and their properties stable over time?

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- **In this presentation we look at domestic data**

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- Disassortative mixing (Fricke *et al.*, 2013)

Basic Network Properties of Layers, 2012

Statistics	U*OVN	U*ST	U*LT	S*ST	Total
# of nodes	595	481	444	38	597
# of edges (net)	2,957	1,103	681	45	3,478
# of selfloops	66	34	24	18	72
Density	0.008	0.005	0.003	0.032	0.010
Avg. undir. distance	2.345	3.097	2.785	2.585	2.325
Out-degree est. exponent	1.93	2.30	2.62	2.56	1.81
Out-degree # datapoints in the tail	450	371	224	13	98
In-degree est. exponent	3.02	2.25	2.98	2.01	2.85
In-degree # datapoints in the tail	205	433	407	26	242
Out-weight est. exponent	1.63	1.75	2.22	1.46	2.02
Out-weight # datapoints in the tail	183	138	81	18	183
In-weight est. exponent	2.11	1.82	1.97	1.35	1.96
In-weight # datapoints in the tail	190	120	141	14	257

U = Unsecured ; S = Secured

OVN = Overnight; ST = Short Term; LT = Long Term

Exponential Random Networks as null models

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- Links a_{ij} are non identical independent Bernoulli variables
- No need to estimate degree distributions, no exact constraint on degree values (microcanonical ensemble)
- Computational trade-off: parameters sometimes difficult to estimate but networks easy to simulate once parameters are obtained

Benefits from an economic viewpoint

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- Strength distribution not a network property, refers to economic structure (size) & behavior (supply / demand)
- Strengths and degrees are positively correlated
- Strengths and degrees distributions are not stable properties of economic networks, instead they are a function of complex economic interaction, which may change abruptly over time

Topological properties of the domestic Italian interbank market

	2008	2009	2010	2011	2012
Largest weak component	631	627	616	612	595
Simulation average	603.0	600.7	594.3	589.7	568.7
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Largest strong component	445	471	469	472	444
Simulation average	330.5	354.9	353.4	364.0	336.8
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Out-degree assortativity	-0.356	-0.383	-0.388	-0.409	-0.429
Simulation average	-0.331	-0.358	-0.363	-0.377	-0.391
(p-values)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
In-degree assortativity	-0.233	-0.234	-0.234	-0.233	-0.219
Simulation average	-0.215	-0.220	-0.219	-0.218	-0.204
(p-values)	(0.000)	(0.002)	(0.001)	(0.001)	(0.001)
Degree reciprocity	0.345	0.381	0.376	0.362	0.345
Simulation average	0.223	0.238	0.231	0.231	0.211
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Undirected triangles	18,111	14,760	14,589	14,592	13,713
Simulation average	23,105	20,216	20,044	19,710	18,968
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table: Unsecured overnight

	2008	2009	2010	2011	2012
Largest weak component	147	137	393	442	436
Simulation average	124.1	119.3	357.3	388.4	384.5
(p-values)	(0.000)	(0.001)	(0.000)	(0.000)	(0.000)
Largest strong component	24	11	65	173	164
Simulation average	21.2	16.4	38.4	113.3	109.5
(p-values)	(0.230)	(0.129)	(0.000)	(0.000)	(0.000)
Out-degree assortativity	-0.218	-0.213	-0.308	-0.460	-0.442
Simulation average	-0.169	-0.153	-0.290	-0.438	-0.416
(p-values)	(0.052)	(0.042)	(0.213)	(0.043)	(0.043)
In-degree assortativity	-0.298	-0.326	-0.575	-0.682	-0.668
Simulation average	-0.270	-0.231	-0.554	-0.632	-0.612
(p-values)	(0.201)	(0.008)	(0.195)	(0.003)	(0.002)
Degree reciprocity	0.061	0.033	0.114	0.413	0.377
Simulation average	0.064	0.046	0.070	0.215	0.197
(p-values)	(0.448)	(0.260)	(0.000)	(0.000)	(0.000)
Undirected triangles	378	273	258	192	168
Simulation average	576.2	459.5	575.0	966.1	887.0
(p-values)	(0.004)	(0.005)	(0.000)	(0.000)	(0.000)

Table: Unsecured long-term

Largest weak component of a Random Fermionic Network Realization

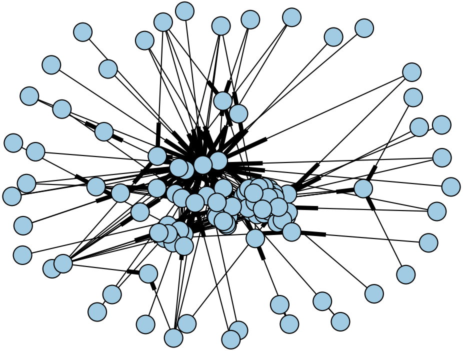

Figure: Unsecured long-term, 2009

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- It's possible to solve the model with both the strength and degree distributions fixed (Bianconi,2008)
- But very difficult to estimate the parameters of this model
- Alternatively, by using Boltzmann weights, we obtain that links in this model are closely approximated by the product of two independent variables (Bernoulli and Poisson): much easier to estimate

Strength-related properties of the Italian Interbank market

	2008	2009	2010	2011	2012
Out-weight assortativity	-0.037	-0.043	-0.114	-0.146	-0.140
Simulation average	-0.187	-0.247	-0.207	-0.201	-0.220
(p-values)	(0.000)	(0.006)	(0.000)	(0.028)	(0.000)
In-weight assortativity	-0.027	-0.027	-0.066	-0.082	-0.055
Simulation average	-0.108	-0.117	-0.111	-0.103	-0.028
(p-values)	(0.002)	(0.015)	(0.059)	(0.365)	(0.159)
Weight reciprocity	0.404	0.167	0.186	0.421	0.286
Simulation average	0.047	0.009	0.016	0.0325	0.031
(p-values)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table: Unsecured overnight

	2008	2009	2010	2011	2012
Out-weight assortativity	-0.005	0.011	0.034	0.017	-0.012
Simulation average	-0.0404	-0.0182	-0.0258	-0.0415	-0.186
(p-values)	(0.081)	(0.204)	(0.345)	(0.129)	(0.015)
In-weight assortativity	-0.004	0.042	0.018	-0.016	-0.050
Simulation average	-0.050	-0.013	-0.010	-0.037	-0.209
(p-values)	(0.027)	(0.288)	(0.427)	(0.236)	(0.050)
Weight reciprocity	0.007	0.000	0.000	0.000	0.017
Simulation average	0.001	0.000	0.001	0.000	0.002
(p-values)	(0.022)	(0.686)	(0.602)	(0.102)	(0.006)

Table: Unsecured short-term

Similarity Analysis of Layers: Measures

We opt for functions $\in [0, 1]$ with no assumptions on the underlining distribution (correlation excluded):

- Jaccard similarity for binary data:

$$J(\mathbf{p}, \mathbf{q}) = \frac{|\mathbf{p} \wedge \mathbf{q}|}{|\mathbf{p} \vee \mathbf{q}|} \tag{1}$$

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- Cosine similarity for valued data:

$$\cos(\theta) = \frac{\mathbf{p} \cdot \mathbf{q}}{||p|| \, ||q||} \tag{2}$$

Steps

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- Permutation test for statistical significance

Jaccard Similarity within Layers

	2008	2009	2010	2011
2009	0.4227*			
2010	0.4393*	0.4767*		
2011	0.2989*	0.2877*	0.4535*	
2012	0.2143*	0.2909*	0.2951*	0.3438*

* 1% significant

Table: Secured short-term (Intersection)

	2008	2009	2010	2011
2009	0.6833*			
2010	0.5674*	0.6531*		
2011	0.5389*	0.5943*	0.7156*	
2012	0.4964*	0.5394*	0.6398*	0.7210*

* 1% significant

Table: Unsecured overnight (Intersection)

Jaccard similarity across Layers

	S*ST	U*OVN	U*LT
U*OVN	0.0096*		
U*LT	0.0112*	0.1906*	
U*ST	0.0067*	0.2060*	0.2255*

* 1% significant.

Table: Jaccard Similarity, 2012 (Union)

	S*ST	U*OVN	U*LT
U*OVN	0.0872*		
U*LT	0.1429*	0.2486*	
U*ST	0.0745*	0.2424*	0.2969*

* 1% significant.

Table: Jaccard Similarity, 2012 (Intersection)

Jaccard Similarity with the Transpose

	S*ST	U*OVN	U*LT
U*OVN	0.0084*		
U*LT	0.0070*	0.1680*	
U*ST	0.0038	0.1803*	0.2622*

* 1% significant.

Table: Jaccard Similarity, 2012 (Union)

	S*ST	U*OVN	U*LT
U*OVN	0.0764*		
U*LT	0.0847*	0.2178*	
U*ST	0.0412	0.2114*	0.3486*

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