

# A flow network analysis of direct balance-sheet contagion in financial networks

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# 1 Financial Networks as Flow Networks

Financial networks, composed of agents connected by financial obligations, arise from four sources:

i) loans and deposits in the interbank money market, ii) 'over-the-counter' trading in assets and derivatives, iii) payment systems; and iv) trade credit, (in the manufacturing sector).

I model them using **Flow Network Theory**, which is a vast and largely applied branch of graph theory. A **flow network** is a **directed and weighted graph** endowed with **source nodes** (no incoming links) and **sink nodes** (no outgoing links).

A **flow** in a flow network is a value assignment to the links of the network subject to two constraints:

a) the value – i.e., the flow – assigned to a link must not exceed its weight – i.e., its capacity, (**capacity constraint**);

b) for each node in the network that is neither a source or a sink node, the total inflow of a node must equal its total outflow, (**flow conservation property**, aka Kirchoff law)

Two models, three applications:

1. **Financial Flow Networks**, useful for modelling the propagation of **solvency shocks** (domino effect) and of **credit crunches** (de-leveraging phases).
2. **Interbank Liquidity Networks** (with Fabio Castiglionesi), we model **liquidity shocks** and consequent interbank **liquidity flows** to evaluate the capability of different interbank networks of providing coverage of **liquidity risk**.

## 2 Financial flow networks

I represent a financial network as a **multisource flow network**, i.e., a directed and connected graph, with some sources and two sinks (more sinks can be added), with links endowed with non-negative capacities, and use it to analyse the **mechanics** of **direct contagion** in financial networks

The network is built using agents' balance sheets. There is a link attached to each balance-sheet item of each agent. There is an **incoming link** into a node for **each asset** owned by the node. Analogously, there is an **outgoing link** for **each liability** of the node. The contagion process, due to an initial insolvency shock, is modeled as a flow that crosses the network.

Formally, a financial flow network is a n-tuple  $N = \{\Omega, A, T, H, L, \Gamma\}$  where:

1.  $\Omega = \{\omega_i\}$  is a set of **nodes** that represent the **financial intermediaries**.
2.  $A = \{a^k\}$ , is a set of **source nodes**, i.e., nodes with no incoming links, that represent the **external assets** held by the members of  $\Omega$ .
3.  $T$  is a **sink**, i.e., a terminal node with no outgoing links, representing the **shareholders** who own the equity of the agents in  $\Omega$ .
4.  $H$  is a **sink** node representing the **households** who hold claims, in the form of deposits and bonds, against the agents in  $\Omega$ .
5.  $L$  is a set of **directed links**  $\{l_{ij}\}$  representing **financial obligations**.

6.  $\Gamma : L \rightarrow R^+$  is a map, called **capacity function**, that associates to each link the value of the corresponding liability.

In modelling flows of value or flows of losses, going from users of funds (external assets) into the portfolios of the providers of funds (shareholders, bondholders and depositors), the direction of the links goes from liabilities into assets. Conversely, in modelling credit crunches, the direction of the links goes from assets into liabilities.

### 3 Direct contagion

**Domino effect:** the system is perturbed by an **external shock**: a drop in the value of some external assets in  $A$ , and the **propagation** of losses, governed by the rules of **limited liability**, **debt priority** and **pro-rata reimbursement** of creditors, is a (legitimate) **flow** that crosses the network

Four sets of results:

1. address issue of **non-uniqueness** (indeterminacy of loss propagation) and embed this in an **algorithm** to compute contagion that controls for possible indeterminacies';

2. **analytical results**, rather than numerical simulations, for the **contagion thresholds** (exposition to systemic risk) for differently shaped networks (namely: complete, regular incomplete, star-shaped and cycle-shaped networks);
3. for generic networks, measures of the effects on contagion of **balance sheet values** such as: capitalization (trivial), leverage, **internal/external debt ratio**  $d/h$ . main finding: the **larger**  $d/h$ , the **more** a network is **exposed** to contagion, both in terms of thresholds and of scope.
4. characterise the **distribution of losses** between **shareholders** (sink  $T$ ) and **debtholder** (sink  $H$ ).

## 4 Cycles and nominal indeterminacy of a propagation

The interdependence of obligations can create problems of indeterminacy due to the joint and simultaneous determination of the losses of the agents that belong to a **strongly connected component** (henceforth SCC) of defaulting agents.

The problem of **non-uniqueness of payment flows** in a financial network was first pointed out by **Eisenberg and Noe** (2001).

Suppose the system contains two nodes, 1 and 2, both defaulting, and each node has nominal liabilities of 1.00 to the other node. In this case, the flow of payments that goes from node 1 towards node 2 depends only on the payments that node 1

receives from node 2, and vice versa, therefore they can reimburse each other with any payment comprises between zero and unity.

I show that:

- a) a clearing payment vector is **not uniquely defined** if and only if it entails **closed SCC's** of insolvent nodes,
- b) the indeterminacy is **confined** to such closed SCC's, and
- c) the emergence of closed SCC's of defaulting nodes in a propagation can be **unambiguously detected**, hence the problem can be controlled for.

These results have a bearing on the algorithms used to compute the domino effect in financial networks.

## 5 Contagion in different network structures

Different networks propagate losses in different fashions. The effects of a shock on a network  $N$  depend on the two elements that form its structure:

- a) the **shape** of the pattern of obligations that constitute the links in  $L$ , and
- b) the values of the headings of the balance sheets of the agents in the network, i.e., the **capacities** of the links in  $L$ .

To evaluate and compare the contagiousness of differently shaped networks, I look at two characteristics of a network: the first and the final thresholds of contagion.

The **first threshold** of contagion of a network  $N$  is the magnitude of the smallest shock that is large enough to cause secondary defaults.

The **final threshold** of contagion of a network is the value of the smallest shock that is capable of inducing the failure of all nodes in the network.

I found very useful a known property of network flows (known as *flow constancy*): the value of the net forward flow that crosses all cuts of  $N$  is constant and equal to the value of the exogenous shock.

### **Results:**

i) in **complete** networks, the first and the last threshold of contagion coincide;

ii) the same applies to **star-shaped** networks **if** the central node is in the initial set of defaults;

iii) in **incomplete** networks where all nodes have the same indegree and outdegree (hence the same degree of centrality), the difference between first and last thresholds grows as connectivity diminishes; such difference is maximal for cycle-shaped networks.

iv) the first (final) thresholds of incomplete regular and cycle-shaped networks are both smaller (larger) than the one of complete networks;

This implies that the class of incomplete regular networks (which includes the cycle-shaped ones), compared to the class of complete networks, is more exposed to contagion due to shocks of small magnitude and scope, and less exposed to the risk of complete system melt-downs.

## 6 Comparison between complete and star-shaped networks:

For the sake of comparability, I assume that:

- i) the total stock of equity,  $E$ , the total external debt,  $H$  and the total internal debt,  $D$ , are the same in  $N^c$  and  $N^s$ .
- ii) all nodes have the same balance-sheet ratios. Under these restrictions the contagion thresholds are:

1. In a complete network  $N^c$ , the first and the final threshold are equal to:

$$\tau'_c = \tau''_c = E + \frac{EH}{D} \frac{n-1}{n}, \quad (1)$$

2. In a star-shaped network  $N^s$ , the first threshold,  $\tau'_s$ , and the final threshold,  $\tau''_s$ , of contagion coincide if the central node is in the set of primary defaults:

$$\tau'_s = \tau''_s = E + \frac{EH}{D} \frac{1}{2}. \quad (2)$$

If the central node  $\omega_c$  is not in the set of primary defaults, the first and the final threshold of contagion of the star-shaped network do not coincide. The first threshold is smaller than  $\tau_c$  while (naturally) the final threshold is larger than  $\tau_c$ .

Dropping assumption (ii) and endowing the central node with the same equity and deposits as the peripheral nodes, the (1) and (2) are the same.

## 7 Value of balance sheets headings and contagion thresholds

All the above characterised contagion thresholds are monotonically increasing in the equity endowments,  $e$ , and in the  $h/d$  **ratio**.

The larger the equity of the members of the network, the higher the contagion thresholds (obviously).

The  $h/d$  **ratio** governs the allocation of the flow of losses, released by defaulting nodes, between external creditors (households) and internal ones, i.e., other nodes in  $\Omega$ . The smaller the  $h/d$  ratio, the smaller the portion of losses that, at each default,

is sent into the sink  $H$ , and the larger the flow of losses that continues to circulate among the nodes in  $\Omega$ , and vice versa.

I show that, **for generic financial flow networks**, the **smaller** the  $h/d$  **ratio**:

1. the **smaller** the **contagion thresholds**, and
2. the **larger** the **scope** of contagion, i.e. the **number of defaults** induced by a shock.

Conversely, a change of the size of internal exposures  $d$  which is accompanied by an equivalent change of the stock of external debt  $h$  (i.e., that leaves the  $h/d$  ratio constant), has no effect on any of the above defined contagion thresholds.

Contrary to what intuition would suggest, neither the total stock of debt,  $h + d$ , nor the leverage ratios – i.e., the debt-to-equity and the asset-to-equity ratios – have any consequence on such thresholds. This means that debt and leverage have no effects on the dynamics of a contagion process in a network  $N$ , while they do have an effect on the expected size of the shocks to which the network is exposed.

## 8 Distribution of losses between debtholders and shareholders (distress contagion)

The **distribution of losses** between **shareholders** (sink  $T$ ) and **debtholder** (sink  $H$ ) depends on both the  $h/d$  ratio and on the shape of the network:

- i) the **smaller**  $h/d$ , the **larger** the losses born by **shareholders** and the **smaller** the losses born by **bondholders** and **depositors**.
- ii) shareholders are better off (smaller losses) with sparse networks, while debtholders are more protected with highly connected (e.g. complete) networks (as long as liquidation costs are negligible)

## 9 Conclusions

1. Flow network theory is a convenient analytical tool for the modelling of financial systems; the above mentioned measures of systemic risk and liquidity risk coverage can be directly applied to any database representing a financial network (such as interbank liquidity networks, data available to central banks)
2. There is no monotonic relation between connectivity and exposition to systemic risk: sparse but highly centralized networks behave like highly connected networks;
3. The structure (shape, capitalization and  $h/d$  ratio) of the networks determine both the exposition to systemic risk as well as the final allocation of losses among the providers of funds, and this should be taken into account for policy purposes;

4. Intra-network obligations are a major source of systemic risk and should be a major concern for authorities: its a risk-sharing device that generates systemic risk and moral hazard. (shadow banking provides the perfect ground for this

mechanism)

The end

Thank you!