Optimal investment portfolio for a robust financial system

Yoshiharu Maeno, NEC
Kenji Nishiguchi, JRI
Satoshi Morinaga, NEC
Hirokazu Matsushima, IISE

Problem

- The financial system of interlinked banks is a backbone of the economy.
- The contagion of bank bankruptcies ensues from a financial crisis.
- What is the optimal investment portfolio of banks to achieve a robust financial system?

ANWSER model

- To solve the problem, the ANWSER model generates random samples of a financial system
- The model simulates the transmission of distress from banks to banks while the market fluctuates.
- The model analyzes the number of bank bankruptcies statistically.

Quantifying contagion

- The ANWSER model inputs four structural parameters.
  - Diversity and risk exposure of investments
  - Denseness and concentration of interbank loans
- The model outputs bankruptcy reproductive ratio.

Balance sheet

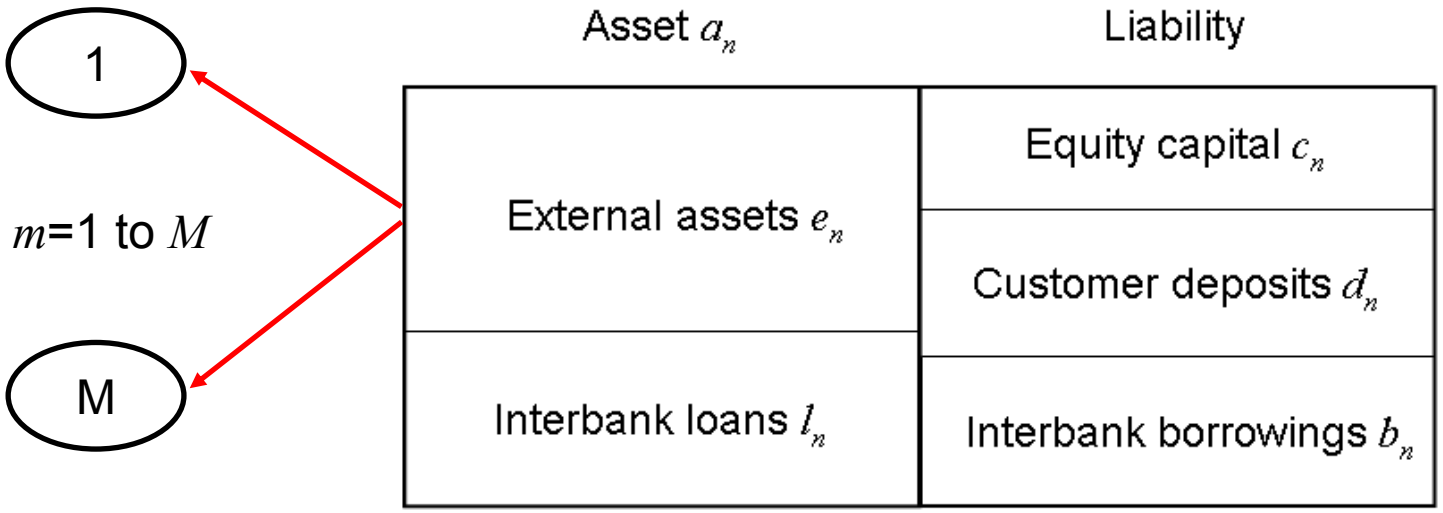
Asset a_n	Liability
External assets e_n	Equity capital c_n
	Customer deposits d_n
Interbank loans l_n	Interbank borrowings b_n

Model of the balance sheet of a bank

γ : equity capital ratio

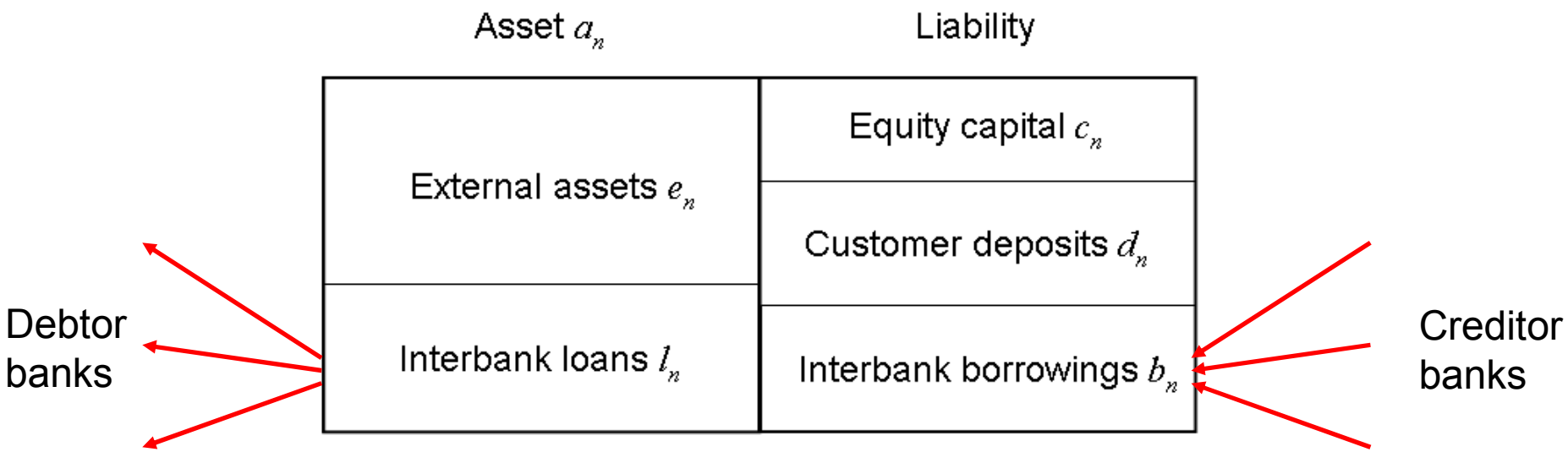
θ : interbank loans as a fraction of the assets

Investment portfolio



The amount of the m-th asset which the n-th bank invests in is X_{nm} as a fraction of the external asset. $\boldsymbol{X}$ describes the investment portfolio of the entire financial system.

Interbank credit network



Banks are interlinked by interbank loans. Creditors own loans while debtors own the corresponding borrowings. The topology of this interbank credit network is $\boldsymbol{T}$.

Simulation

- Distress transmission
  - The external assets are impaired when the asset price falls in the market.
  - The interbank loans are impaired when the debtor banks go bankrupt.
- Bankruptcy condition
  - The bank goes bankrupt if the equity capital can not absorb these losses.

Structural parameter

Diversity δ and risk exposure ε of an investment portfolio

$$\delta(\boldsymbol{X}) = \frac{1}{N(N-1)} \sum_{n \neq n'} \frac{1}{M} \sum_{m=1}^{M} |X_{nm} - X_{n'm}|.$$

$$\varepsilon(\boldsymbol{X}) = \frac{1}{N} \sum_{m=1}^{M} |\sum_{n=1}^{N} X_{nm} - \frac{1}{M}|.$$

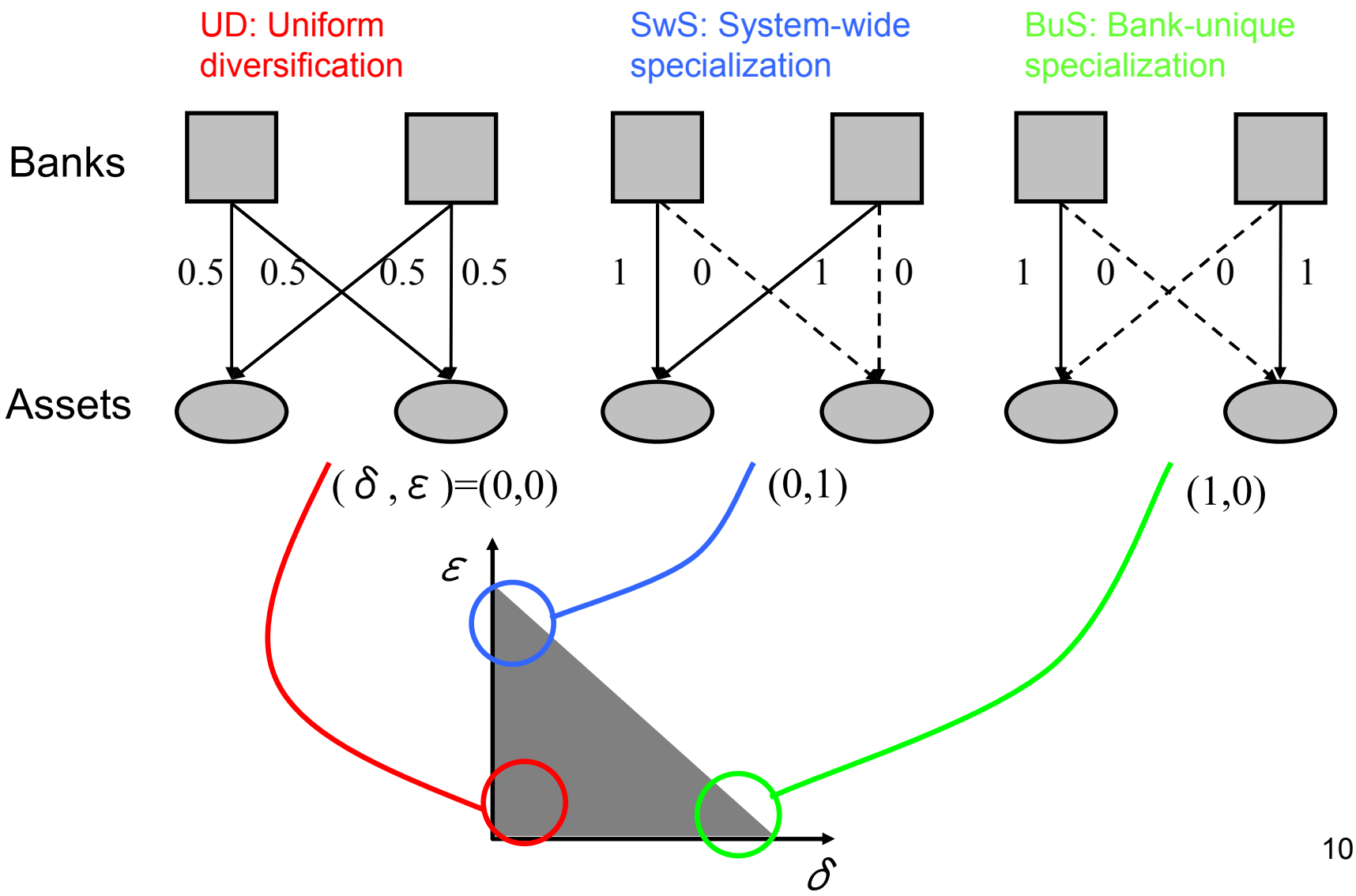
Denseness κ and concentration ρ_5 of a credit network

$$\kappa(\boldsymbol{T}) = \frac{1}{N(N-1)} \sum_{n \neq n'} T_{nn'}.$$

$$\rho_5(r) = \frac{\sum_{n=n_1,n_2,n_3,n_4,n_5} l_n}{\sum_{n=1}^{N} l_n}.$$

The definition is given by N. Beale *et al*., PNAS **108** 12647 (2011).

Diversity and exposure



Reproductive ratio

- Bankruptcy reproductive ratio A is the strength of a subsequent chain effect of contagion.
- Risk landscape is A as a function of the diversity, risk exposure, denseness, and concentration.

$$A(\kappa, \rho_5, \delta, \varepsilon) = \frac{|\boldsymbol{F}_\infty|}{|\boldsymbol{F}_0|}$$

Final number of bankruptcies after the contagion comes to an end

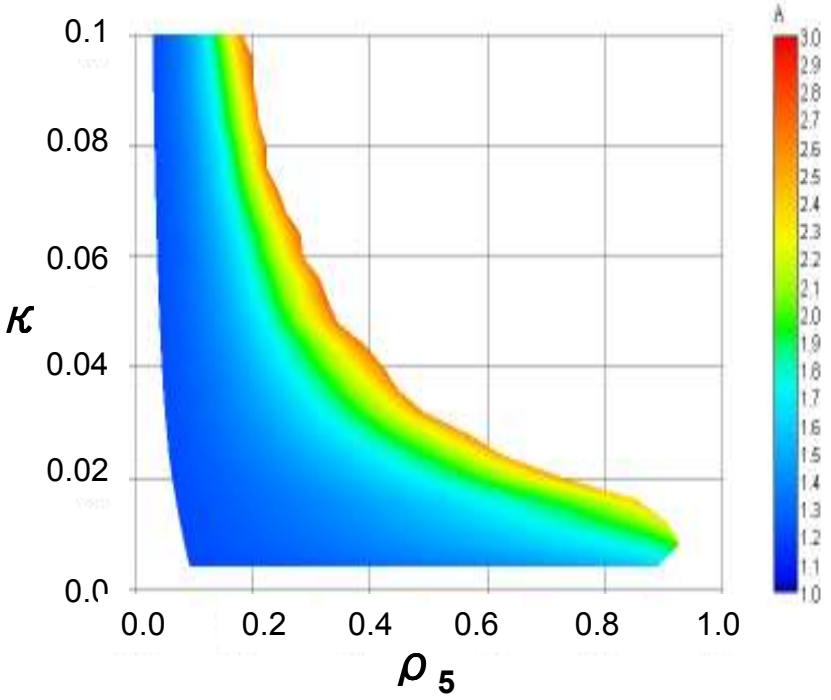
Initial number of bankruptcies immediately after the asset price fluctuates

The definition is presented by Y. Maeno *et al*., IEEE CIFEr (2013).

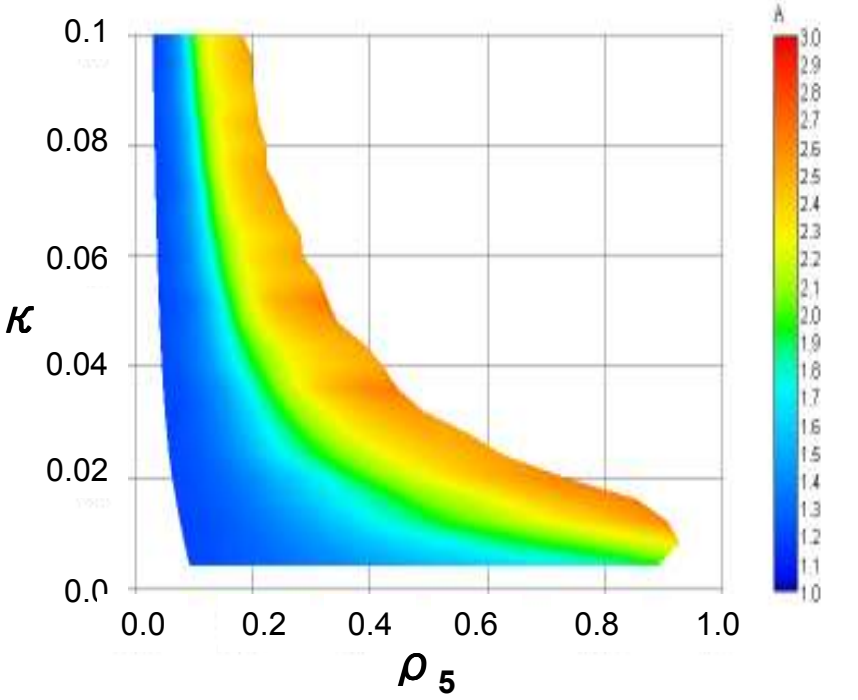
Risk landscape $A(\rho_5, \kappa)$

N=500, M=2, δ=0.33, ε=0.02, γ=0.07, θ=0.1

Average



The 999-th 1000-quantile



"Too concentrated" systems and "too connected" systems are dangerous.

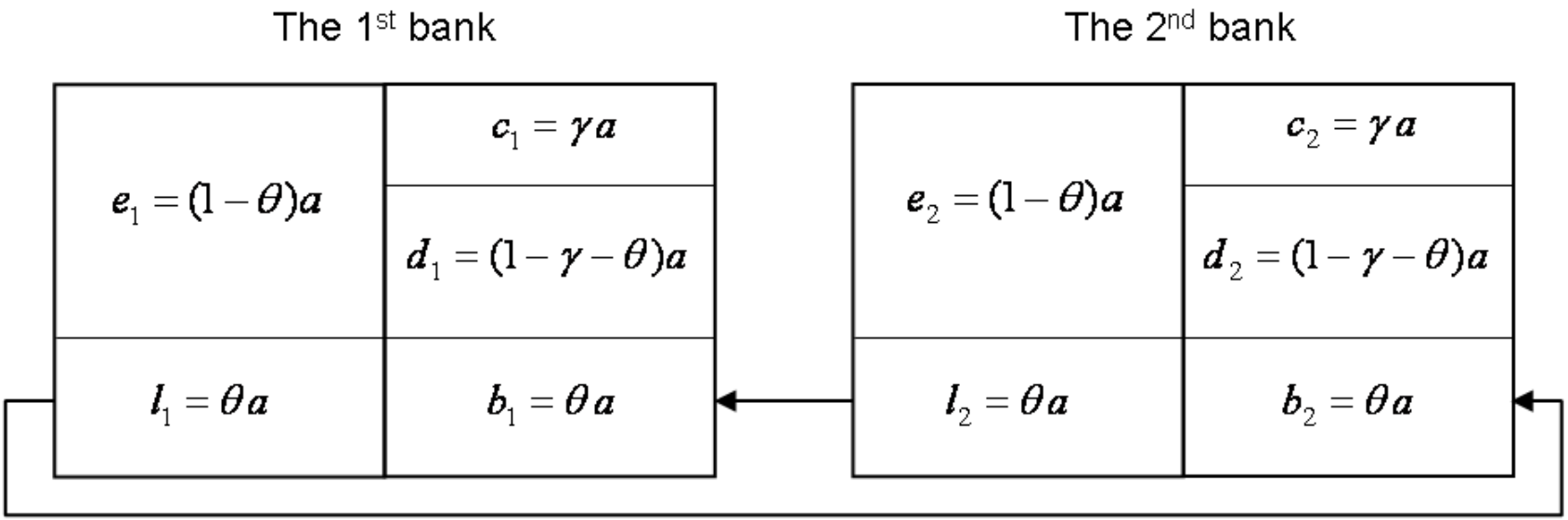
The landscape for the worst case is almost the same as for the average.

Analytical solution

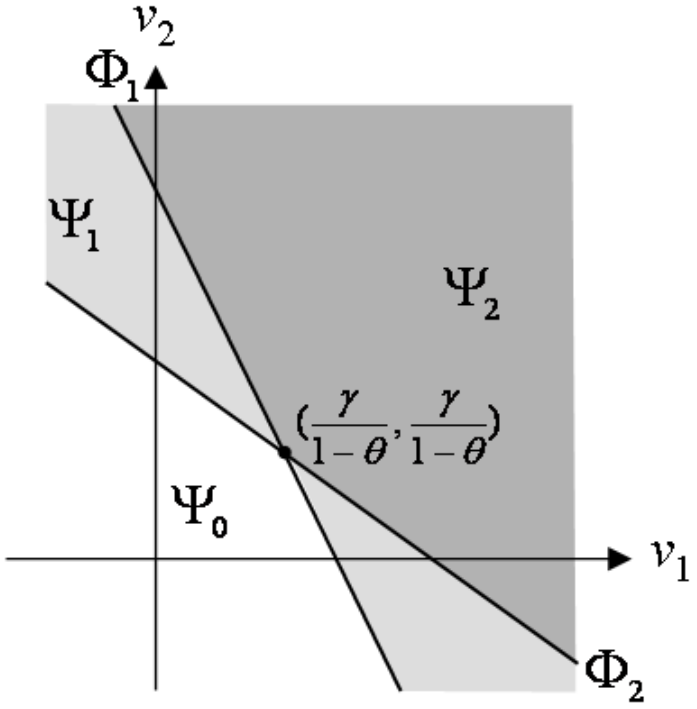
The number of banks N=2.

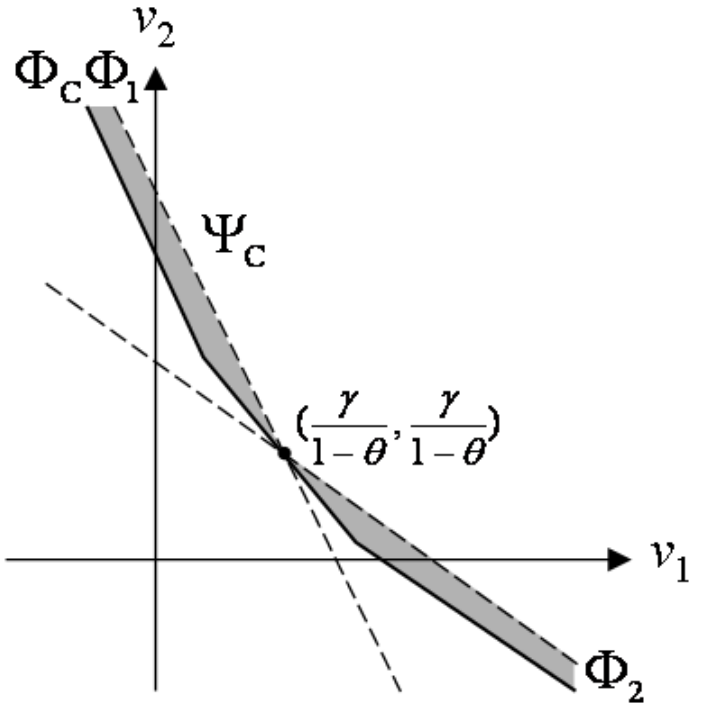
The number of assets M=2.

Asset price fluctuation: exponential distribution



Bankruptcy condition





$$\begin{cases} \Phi_1: \ \gamma = (1-\theta)\{X_{11}v_1 + (1-X_{11})v_2\} \\ \Phi_2: \ \gamma = (1-\theta)\{X_{21}v_1 + (1-X_{21})v_2\} \end{cases}.$$

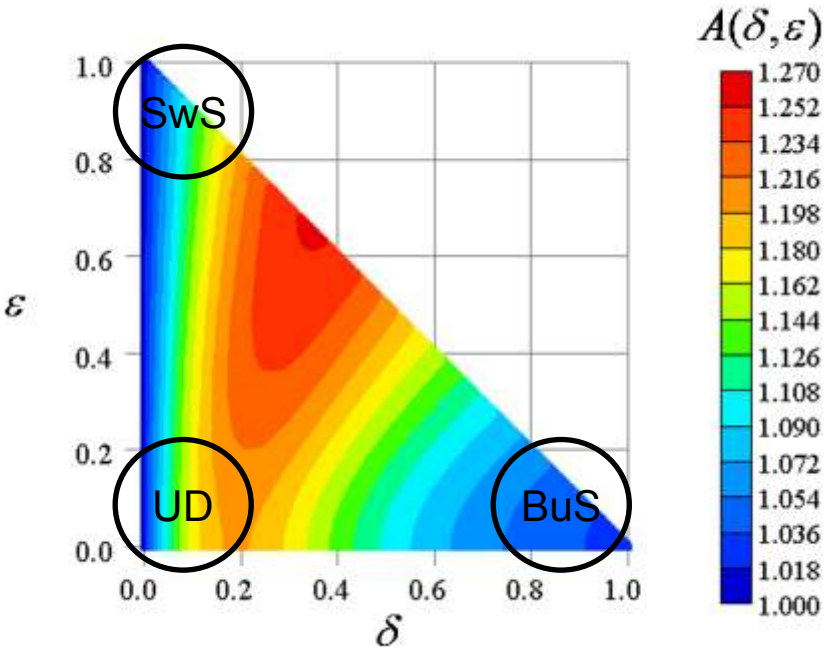
$$\Phi_C: \begin{cases} \gamma = \min((1-\theta)\{X_{21}v_1 + (1-X_{21})v_2\} - \gamma, \theta) \\ \quad +(1-\theta)\{X_{11}v_1 + (1-X_{11})v_2\} \text{ if } v_1 \le \frac{\gamma}{1-\theta} \\ \gamma = \min((1-\theta)\{X_{11}v_1 + (1-X_{11})v_2\} - \gamma, \theta) \\ \quad +(1-\theta)\{X_{21}v_1 + (1-X_{21})v_2\} \text{ if } v_1 > \frac{\gamma}{1-\theta} \end{cases}.$$

$$p(|\boldsymbol{F}_0| = 1, |\boldsymbol{F}_1| = 2) = \iint_{\Psi_C} P(v_1)P(v_2)\mathrm{d}v_1\mathrm{d}v_2.$$

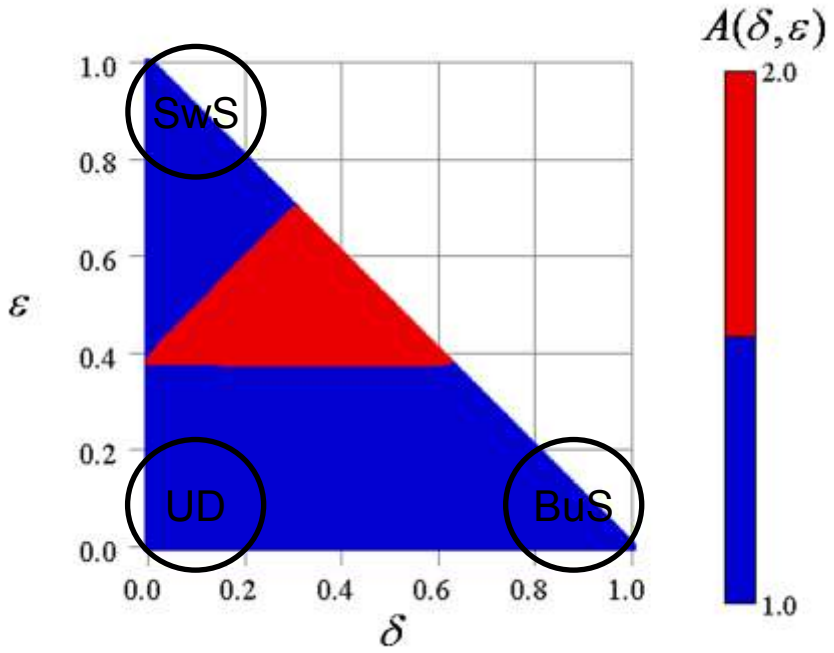
Risk landscape $A(\delta, \varepsilon)$

N=2, M=2, γ =0.07, θ =0.1

Average



The 999-th 1000-quantile



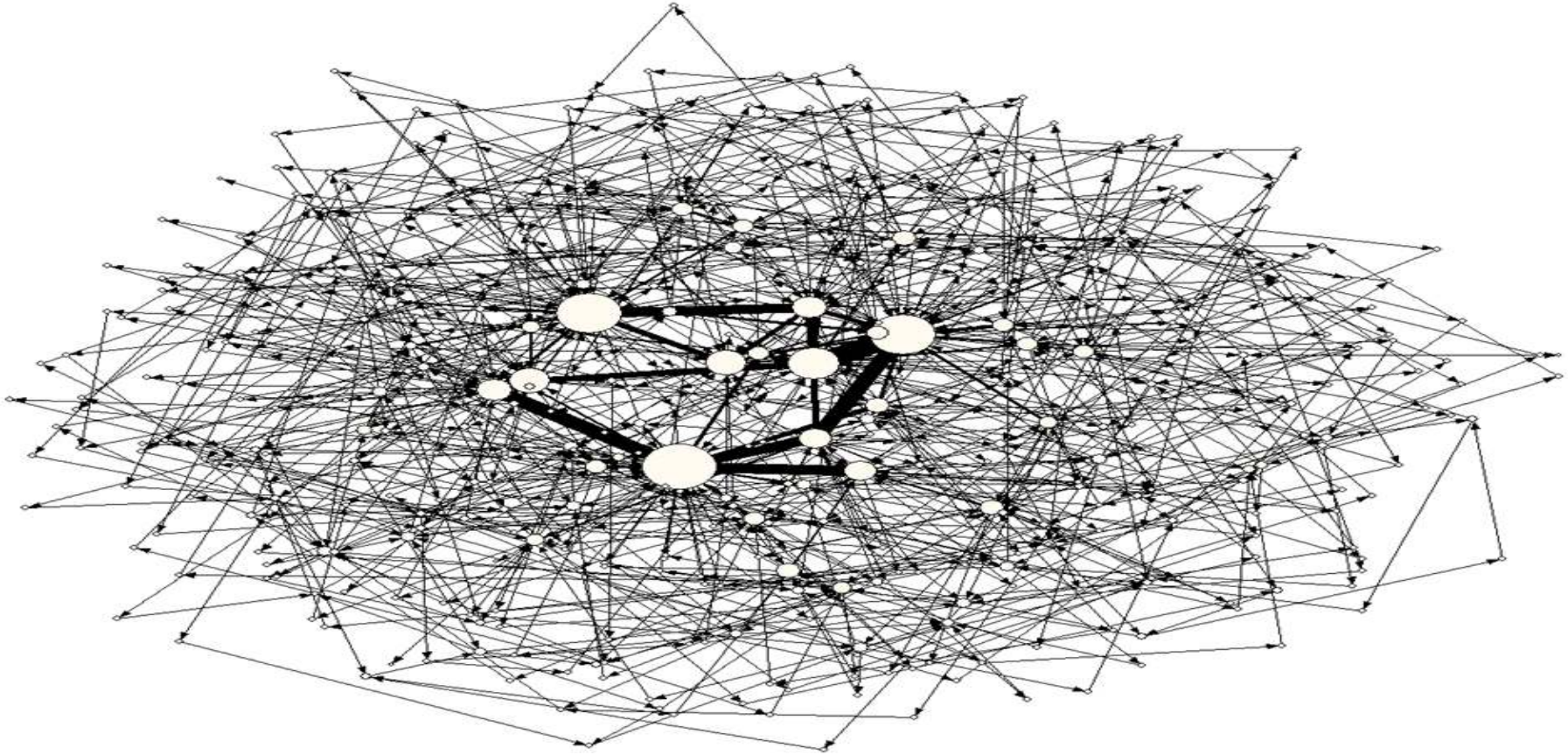
Large diversity is effective in eliminating the risk of contagion.

A bank-unique specialization is superior to a uniform diversification.

Numerical simulation

The number of banks N=500, The number of assets M=2.

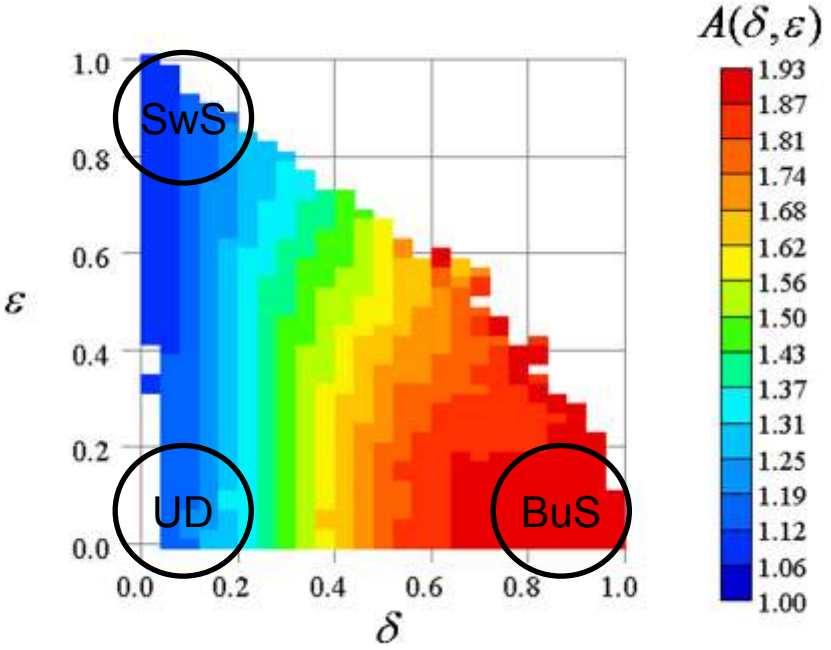
Asset price fluctuation: Student's t-distribution (df=1.5)



Risk landscape $A(\delta, \varepsilon)$

N=500, M=2, κ=0.05, ρ_5=0.25, γ=0.07, θ=0.1

Average



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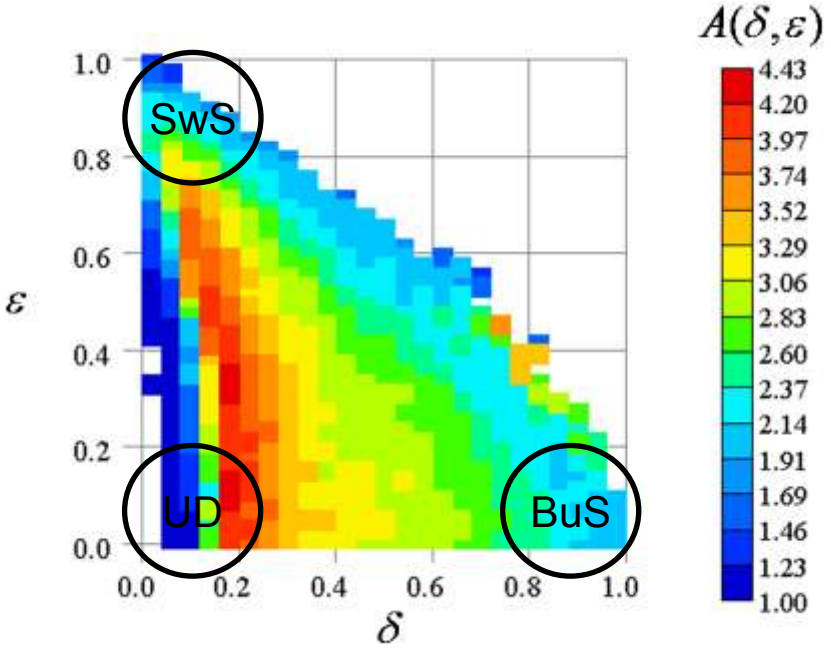
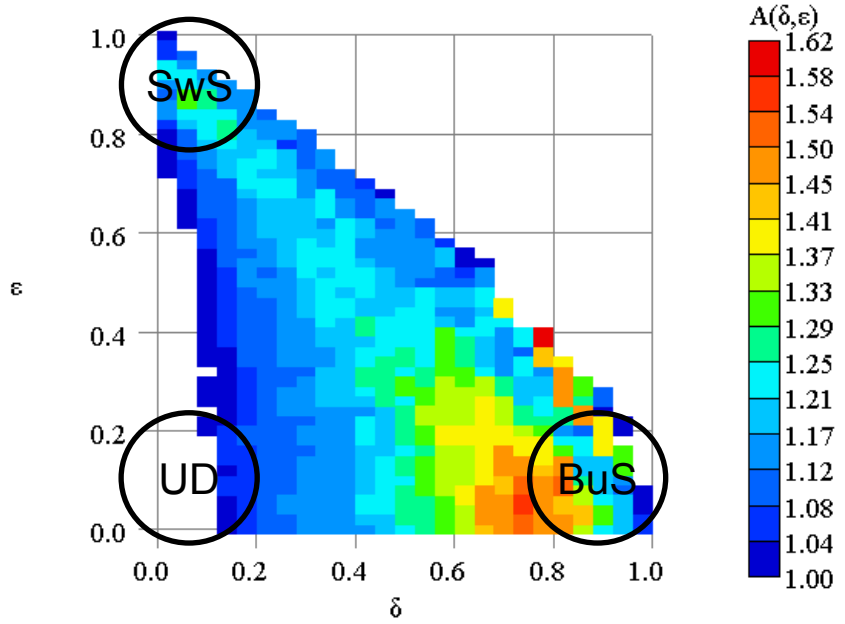


Large diversity is essential in removing contagion in the worst case.

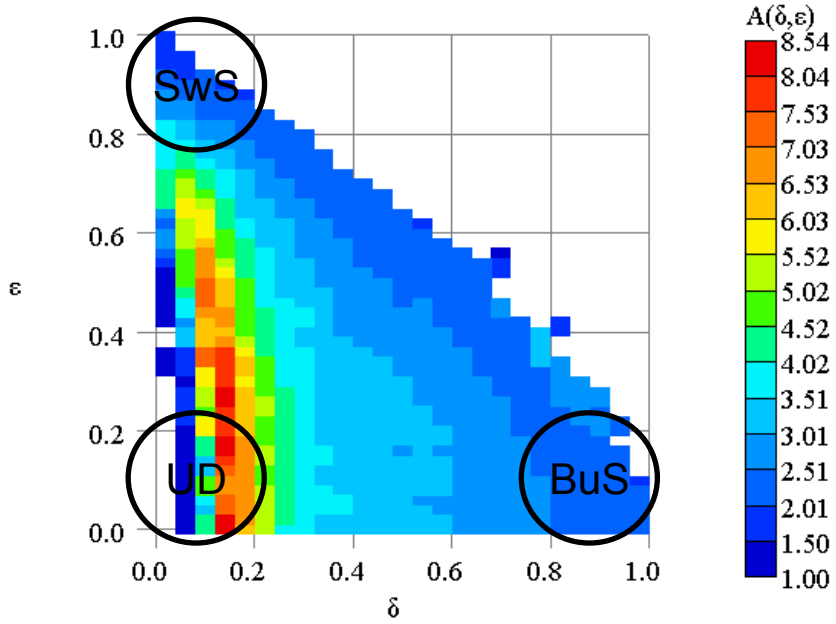
A bank-unique specialization is superior to a uniform diversification.

Dependence on ρ_5

N=500, M=2, κ =0.05, ρ_5=0.05,
γ =0.07, θ =0.1



N=500, M=2, κ =0.05, ρ_5=0.5,
γ =0.07, θ =0.1

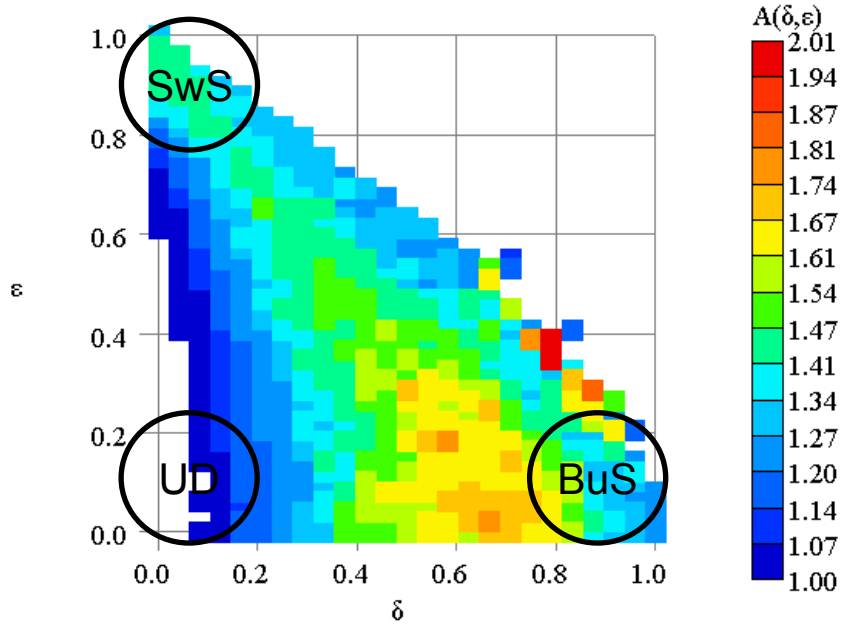


Contagion gets stronger as the system becomes more concentrated.

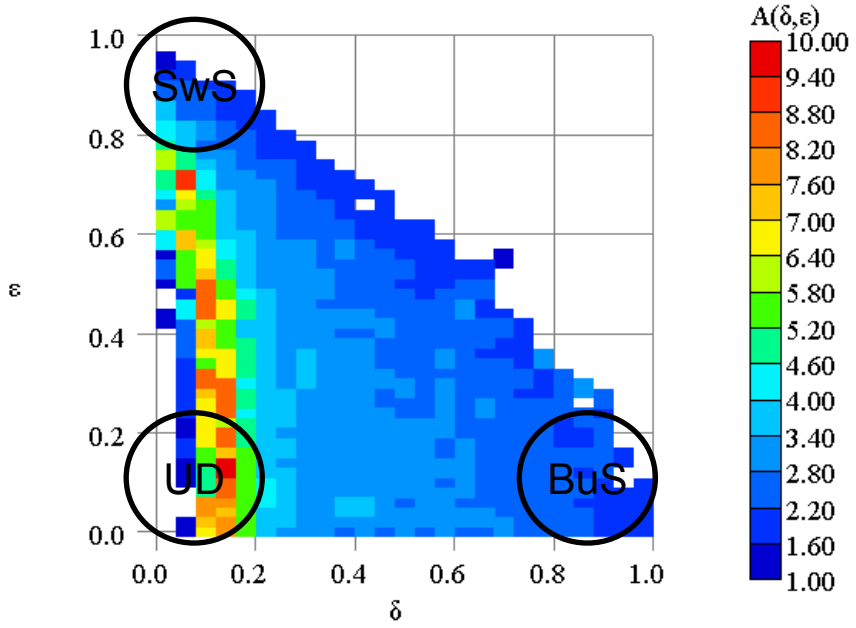
A uniform diversification becomes much more dangerous.

Dependence on κ

N=500, M=2, κ =0.01, ρ_5=0.25,
γ =0.07, θ =0.1



N=500, M=2, κ =0.1, ρ_5=0.25,
γ =0.07, θ =0.1



Contagion gets stronger as the system becomes more connected.

A uniform diversification becomes much more dangerous.

Conclusion

- Given structural parameters of investments and interbank loans, the bankruptcy reproductive ratio is calculated by the ANWSER model.
- Diversity is an essential structural parameter.
- Uniform diversification is a conventional practice, but not optimal for a robust financial system.

Future perspective

- From Financial Engineering to Systems Economics
  - How does the economy work when individual financial institutions take an egoistic self-defense action?
  - Can we predict the real-time risk hidden in the complexity of the global economy?
  - How should we regulate individual financial institutions to maximize public economic interests?