Risk-Taking in Financial Networks

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Abstract

Bankruptcy cascades in financial networks can be amplified by portfolio choices. From banks' private perspective, limited liability and a risky payoff structure of illiquid assets create a risk-taking incentive, while the externalities of bankruptcy on the remainder of the network are ignored. Therefore, a Pareto-inefficient generalized breakdown is possible, especially when asset returns are high and shock probabilities low. This systemic risk can be contained with a bailout fund.

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1 Introduction

In financial crises, trouble in some market segments or institutions spills over quickly to the entire banking system. The events of 2007-2009 provided two powerful illustrations for this financial contagion, after the losses in the United States' subprime mortgage market in 2007 and after the failure of Lehman Brothers on September 15th, 2008.

Financial contagion is often explained by direct links. As banks are related to each other by debt contracts, a shock to a single one of them can be sufficient to trigger a bankruptcy cascade through repeated defaults. Allen and Gale (2000) first formalized

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this argument with a model in which both the financial network and banks' actions are exogenous.

Their approach has been extended in a recent paper by Caballero and Simsek (2013), who endogeneize banks' actions. Caballero and Simsek study a circular network of banks linked by short-term debt claims, in which it is common knowledge that a particular bank will be hit by a liquidity shock in the immediate future. This shock triggers a bankruptcy cascade. Banks knowing they will be directly or indirectly affected sell their illiquid assets to unaffected banks and thereby limit the cascade's length. However, when there is uncertainty about the network position of the distressed bank, the same type of portfolio choices amplifies the shock. All banks assume their counterparty may default and try to sell assets to cope with this possibility. A flight to liquidity triggers a fire sale, reducing aggregate liquidity and causing more bankruptcies.

I generalize this model by assuming banks have to decide on their portfolio composition before knowing for sure whether a shock will hit one bank in the network. In Caballero and Simsek's model, banks' initial portfolios are crucial for amplification: the less illiquid assets they hold, the less they are affected by a fire sale. It seems realistic that banks chose these initial portfolios without knowing for sure whether there would be a shock to the network. My model analyzes their ex ante decisions and shows how they create a new, powerful amplification effect.

In the model, illiquid assets have a higher return than cash but cannot be used to settle debts in a liquidity crisis. Combined with limited liability for bank managers, this payoff structure creates a risk-taking incentive, as has been shown by a large theoretical literature (Jensen and Meckling (1976), Stiglitz and Weiss (1981), Diamond and Rajan (2011))[1].

Embedded in a network model, risk-taking generates new and interesting results. Banks tend to enter a crisis episode with illiquid portfolios because they neglect the externality their choices impose on the remainder of the network. The risk-taking incentive is stronger when asset returns are high and shock probabilities are low, so that systemic risk increases in good times. When banks cannot contain the shock by themselves, a

[1] Risk-taking also has some empirical support (see, for example, Akerlof and Romer (1993) or He, Khang and Krishnamurthy (2010)).

bailout fund with mandatory contributions can achieve a Pareto-improvement.

As Allen and Gale and Caballero and Simsek, I take the financial network as given and do not consider banks' incentives to engage in it. Since Allen and Gale's original contribution, a large and growing literature has studied network formation. The typical trade-off banks face in this literature is described, among others, by Brusco and Castiglionesi (2007): interbank linkages are a way to mutualize idiosyncratic shocks, but also an exposure to counterparty risk. In their model, the first effect dominates, so that a network is a Pareto-improvement over financial autarky even though it suffers periodic crises. Other studies focus on inefficiencies in network formation. Zawadowski (2013) shows banks may underinsure against counterparty risk, because insurance is costly and its stability benefits for the overall network are not internalized. Acemoglu, Ozdaglar and Tahbaz-Salehi (2013) prove that even if banks can make loan offers conditional on their counterparty's exposure to the interbank market, they still engage in networks that are too vulnerable to contagion from the perspective of a social planner.

Even though network formation is clearly interesting and important, it is outside of the scope of my paper. Instead, I analyze banks' portfolio choices in a given network, an issue which I believe to be of independent interest.

2 The model

My setup closely follows Caballero and Simsek. There are three periods, $t \in \{0, 1, 2\}$, and n identical banks $(b_0, b_1, ..., b_{n-1})$, with balance sheet given in Figure 1.

Figure 1: Balance sheet of bank b_i at $t = 0$

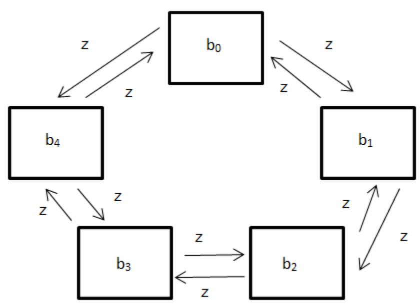
Assets		Liabilities	
z	Debt claim on b_{i-1}	z	Debt owed to b_{i-1}
z	Debt claim on b_{i+1}	z	Debt owed to b_{i+1}
y	Cash	$y + p(1-y)$	Equity
$p(1-y)$	Assets		

Debt is due at t = 1. b_0 is connected to b_{n-1} and b_1, and b_{n-1} is connected to b_{n-2} and b_0.

Every bank initially owns $y \in (0, 1)$ units of cash (the numeraire) and $1 - y$ units of asset. Cash can be stored with a return of 1 across periods, while one unit of the asset pays $R > 1$ units of cash at $t = 2$.

Banks maximize the expected value of their equity in the last period by choosing one out of two portfolio actions at $t = 0$. They can either use all their cash to buy more assets and hold on to the ones they have (this action is denoted by B, for "buy") or sell all their assets for cash (denoted by S, for "sell"). As in Caballero and Simsek, limiting actions to a binary choice is not restrictive. Allowing banks to sell only a part of their assets can change some actions (for instance, a bank with a small liquidity need would only sell part of its assets and hold on to the rest) but does not affect qualitative conclusions. New assets are supplied inelastically at a price of 1. As long as the marginal buyer is a bank, the asset price will therefore be $p = 1$. However, when banks cannot absorb the entire asset supply (because too many banks chose S and too few B), some assets must be sold to outside agents with a lower valuation and the price falls to $p_{scrap} < 1$.

Figure 2: Bank network for $n = 5$



Banks are connected by a circular network of debt claims due at $t = 1$ (illustrated in Figure 2). Every bank has a claim of z on and owes z to its forward and backward neighbor. For most of the discussion, the network is assumed to be common knowledge. I will also eventually consider the consequences of introducing network uncertainty as in Caballero and Simsek. However, it turns out that this does not affect my results, as the new amplification mechanism created by risk-taking dominates.

Assets are illiquid at $t = 1$ and cannot be used to settle debts. The absence of an asset market in this period is crucial, because it creates a return-liquidity trade-off. In normal times, this is not an issue, as all banks can pay their creditors with the cash they receive from their debtors. However, with probability $\pi \in (0, 1)$, a local shock hits the network and b_0 has to pay θ units of cash to an outside agent. Thus, b_0 may want to hold some cash, and if it defaults on some of its debts, other banks may want to do so as well.

Debt payments are perfectly enforced. Therefore, when a bank has a sufficient amount of cash to pay (which is for example always the case when the local shock does not hit), it must do so. This even applies when its counterparty is bankrupt. A bank is bankrupt if its cash holdings are insufficient to pay back all its debts. It then has an equity value of 0, reflecting limited liability. When a bankrupt bank receives any payments, they are equally split among its creditors. The outside payment is an exception to this rule and is assumed to be senior. This is not crucial for any of the results, but simplifies calculations.

Two technical assumptions complete the model.

$$\theta > 2z \tag{1}$$

$$ny > 2 \tag{2}$$

Assumption (1) implies the outside claim is large enough to absorb all cash b_0 may get from its debtors. It is made for convenience and does not affect results. Assumption (2) ensures aggregate liquidity is high enough for buying banks to be able to absorb all assets when there are not more than two sellers. As the model is aimed at large networks, this is not restrictive.

Banks only take decisions in the initial period $t = 0$. Their action profile will be a (Nash) equilibrium if, taking the asset price p as given, no bank has a profitable deviation against the others' actions, and the price is such that the asset market clears.

I show in Appendix A2 that this model replicates Caballero and Simsek[2] when $\pi = 1$. It is however convenient to interpret the cases $\pi = 1$ and $\pi < 1$ somewhat differently. When $\pi = 1$, the model describes a crisis episode in which banks know that the network is under stress. The shock triggers a bankruptcy cascade, and when network positions are uncertain, all banks think they may be hit, try to sell their assets and trigger a fire

[2] In fact, when $\pi = 1$, both models are identical up to a small change in the network structure. The reason for this change, which does not affect results neither in the original nor in my model, is explained as well in Appendix A2.

sale that amplifies the cascade. When $\pi < 1$, it is not sure a priori whether the network will be shocked or not. The initial period may then be thought of as the period preceding a potential Caballero and Simsek-type fire sale crisis (the fire sale being proxied in my model by the assumption that the asset is illiquid in the intermediate period). In that sense, my model is complementary to the one of Caballero and Simsek.

3 Risk-taking and network equilibria

To find the equilibria, I first determine banks' optimal actions taking liquidity needs and prices as given. Then, I endogeneize liquidity needs by looking at the network, and finally the asset price by imposing market clearing.

3.1 Banks' optimal actions

The optimal choice of a bank depends on the amount of cash it needs in the intermediate period in case the local shock hits[3]. Two different situations can arise.

Case 1 When the local shock hits, the bank does not need cash. This happens when its debtors pay back in full and it does not face an outside liquidity need.

In this case, the banks' decision problem is trivial. It always chooses B, because the return on assets exceeds the return on cash.

Case 2 When the local shock hits, the bank needs an amount of cash $L > 0$ to pay back its creditors. This happens when its debtors do not pay back in full or when it faces an outside liquidity need.

Now, there is a trade-off: action B leads to bankruptcy if the shock hits, but gives a higher return if it does not. Denote $l(p) = (1 - y) p + y$ the amount of cash the bank can get by selling all its assets. The expected equity value at $t = 2$ when choosing S is

$$\pi \max(0, l(p) - L) + (1 - \pi) l(p)$$

When choosing B, it is

$$(1 - \pi) R \frac{l(p)}{p}$$

This payoff structure provides an incentive for risk-taking. To see this, consider the two sub-cases that arise here.

[3]Recall that if the shock does not hit, no bank needs cash in the intermediary period.

Case 2.1 No matter which action the bank chooses, it goes bankrupt when the shock hits: $L > l(p)$.

Then, B is always a better choice than S, because

$$\underbrace{(1-\pi) R \frac{l(p)}{p}}_{\text{Payoff of } B} \quad > \quad \underbrace{(1-\pi) l(p)}_{\text{Payoff of } S}$$

This inequality always holds, because $0 < p \leq 1$ and $R > 1$. From the perspective of bank managers, the worst-case scenario payoff of both actions is the same, so they maximize the payoff in the best-case scenario by choosing B. From the perspective of the rest of the network, this action is more risky, because the losses after a potential bankruptcy are larger. I will therefore sometimes refer to B as the gambling action (because its payoff has a larger variance and it increases the risk for the bank's counterparties) and to S as the safe option (because it has a lower variance and limits spillovers).

As gambling takes place at the expense of banks' creditors, one can argue the latter should anticipate it and adapt lending contracts in response. As I consider an exogenous network, I abstract from this issue. Acemoglu, Ozdaglar and Tahbaz-Salehi (2013) show that even when banks can discipline their counterparties by loans that are conditional on interbank market exposure, the equilibrium network can still be too connected. Analyzing whether loans conditional on the counterparty's portfolio choice would similarly fail to eliminate risk-taking in equilibrium is beyond the scope of my paper.

Case 2.2 The bank can avoid bankruptcy by choosing S: $L \leq l(p)$.

This is the only case in which a bank may choose S. It will do so if[4]

$$\pi (l(p) - L) + (1-\pi) l(p) \geq (1-\pi) R \frac{l(p)}{p}$$

This holds if and only if

$$\frac{(1-\pi) R}{p} < 1 \tag{3}$$

and

$$l(p) \geq \frac{\pi p}{p - (1-\pi) R} L \tag{4}$$

[4] I assume an indifferent bank chooses S. This does not matter for results.

Both conditions have a clear interpretation. The left-hand side of (3), $\frac{(1-\pi)R}{p}$, is the lowest possible return to investing one unit of cash in the asset. This strategy may pay nothing in case the shock hits, but it always guarantees a return of $\frac{R}{p}$ when the shock does not hit, which happens with probability $1-\pi$. The right-hand side, 1, is the highest possible return to holding one unit of cash. So, when the lowest possible return to investing in the asset exceeds the highest possible return to cash, it is clear the bank will always gamble, no matter what its liquidity need or network position is.

When the return condition (3) does not hold, conditional liquidity needs (which depend on the network position) matter. The bank chooses the safe action S if the amount of cash it can rescue after a shock is larger than a lower bound given by (4). This threshold is decreasing in π, the likelihood of the shock.

Note that (3) and (4) imply $L \leq l(p)$: a bank that finds it optimal to avoid bankruptcy is also able to do so[5]. Also, there are no partial repayments. A bank pays back either its entire liquidity need or nothing at all. Lemma 1 summarizes banks' optimal actions.

Lemma 1. A bank with conditional liquidity need L chooses S if and only if

(i) $\quad L > 0$

(ii) $\quad \frac{(1-\pi)R}{p} < 1$

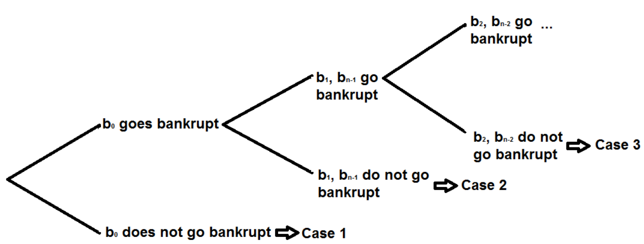
(iii) $\quad l(p) \geq \frac{\pi p}{p-(1-\pi)R} L$

In all other cases, the bank chooses B.

3.2 Network equilibria for a given asset price

It is easiest to find all possible equilibria by considering all possible outcomes. I therefore distinguish a sequence of cases, illustrated in Figure 3.

Figure 3: Case distinction for network equilibria



[5] I have shown the contrapositive in Case 2.1: when S cannot avoid bankruptcy, B is preferred to S.

Figure 3 exhausts all possible scenarios because the network's symmetry implies that for $k \geq 1$, b_k and b_{n-k} must always have the same outcome. Also, b_k and b_{n-k} can only be bankrupt if b_0, b_1, ... b_{k-1}, b_{n-k+1}, ..., b_{n-1} are bankrupt as well (a formal proof of both claims is given in Appendix A1).

Case 1 Conditional on the shock occurring, b_0 does not go bankrupt.

Because of the outside claim, b_0 always has a positive liquidity need when the shock hits. It can therefore only be solvent by choosing S. Moreover, its solvency implies all other banks do not need liquidity and choose B. Under which conditions does this occur? Clearly, no restrictions are needed to make banks other than b_0 choose B. b_0 itself has a liquidity need of $\theta + 2z - 2z = \theta$ (as all other banks, it receives the full value of its short-term claims). According to Lemma 1, it therefore chooses S if

(∗) $\frac{(1-\pi)R}{p} < 1$

(∗) $l(p) \geq \frac{\pi p}{p-(1-\pi)R}\theta$

Case 2 Conditional on the shock occurring, b_0 goes bankrupt, but b_1 does not[6].

All other banks do not need liquidity, remain solvent and choose B. b_1 and b_{n-1} honor their debt payment, so the (conditional) liquidity need of b_0 is still θ. b_0 is bankrupt and must therefore have chosen B. By Lemma 1, this is optimal if $\frac{(1-\pi)R}{p} \geq 1$ or $l(p) < \frac{\pi p}{p-(1-\pi)R}\theta$.

After the shock, b_0 is bankrupt and has no resources. Assumption (1) implies all payments it receives from the solvent bank b_1 go to the outside creditor. b_1 therefore does not receive anything from b_0, but must honor its payments to that bank in order to stay solvent: it has a positive liquidity need of $2z - z = z$. It chooses S to cover this need if $\frac{(1-\pi)R}{p} < 1$ and $l(p) \geq \frac{\pi p}{p-(1-\pi)R}z$.

Summarizing, the necessary conditions for Case 2 are therefore

(∗) $\frac{(1-\pi)R}{p} < 1$

(∗) $l(p) \geq \frac{\pi p}{p-(1-\pi)R}z$

(∗) $l(p) < \frac{\pi p}{p-(1-\pi)R}\theta$

The two last inequalities have an easy interpretation. When the return condition (3)

[6] For brevity, I only consider one side of the symmetric network. Every statement on bank b_k ($k \geq 1$) applies as well to bank b_{n-k}.

holds, a bank chooses S if it gets enough cash to rescue a large part of it after the shock hits. The directly hit b_0 has a higher liquidity need than b_1, so its threshold value is higher. This is why here, b_0 finds it optimal to gamble while b_1 does not.

Case 3 Conditional on the shock occurring, b_0 and b_1 go bankrupt, but b_2 does not. All other banks do not need liquidity and choose B. b_0 needs to pay θ to the outside agent and $2z$ to b_1 and b_{n-1}. The latter banks are bankrupt and therefore a priori unable to pay back anything. However, the cash b_0 would pay to b_1 and b_{n-1} would not disappear: b_1 and b_{n-1} would equally split the received amount among their creditors. So, paying z to b_1 and z to b_{n-1}, b_0 would get back $\frac{z}{2}+\frac{z}{2}=z$. Its liquidity need is therefore $\theta+2z-z=\theta+z$.
b_1 is fully repaid by the solvent b_2 and knows that any payments made to b_0 will be absorbed by the outside agent. Therefore, its liquidity need is $2z-z=z$. b_1 going bankrupt implies it optimally chooses B. This can only be the case if $\frac{(1-\pi)R}{p}<1$ and $l(p)\geq\frac{\pi p}{p-(1-\pi)R}z$. These conditions obviously imply as well the necessary conditions for b_0 to go bankrupt, as b_0 has a higher liquidity need than b_1.
Finally, consider b_2. It is fully repaid by b_3 and knows that when it pays something to b_1, it will get back half of it. b_2's liquidity need is therefore $2z-z-\frac{z}{2}=\frac{z}{2}$. To avoid bankruptcy, it must choose S, which is only possible if $\frac{(1-\pi)R}{p}<1$ and $l(p)\geq\frac{\pi p}{p-(1-\pi)R}\frac{z}{2}$.
Summarizing, the necessary conditions for Case 3 are

(∗) $\frac{(1-\pi)R}{p}<1$

(∗) $l(p)\geq\frac{\pi p}{p-(1-\pi)R}\frac{z}{2}$

(∗) $l(p)<\frac{\pi p}{p-(1-\pi)R}z$

b_1 and b_2 make different choices because b_2 is closer to the safe part of the network and therefore has a smaller liquidity need. However, the safety of some part of the network is due to b_2's choice of S. This hints at the existence of multiple equilibria.

Case 4 Conditional on the shock occurring, b_0, b_1 and b_2 go bankrupt, but b_3 does not. All other banks do not need liquidity and choose B. b_2 is exactly in the same situation as in Case 3: it has a solvent and a bankrupt neighbor which are different from b_0 and therefore a liquidity need of $\frac{z}{2}$. As it goes bankrupt, it must choose B. b_3 also has a solvent and a bankrupt neighbor different from b_0 and therefore the same liquidity need of $\frac{z}{2}$, but must choose S. This is a contradiction: two banks with the same decision problem cannot choose different actions. Case 4 does not exist. It is straightforward to see this argument extends to any other case where for $k \geq 2$, a bank b_k is bankrupt and a bank b_{k+1} solvent.

There is then only one case left to consider, the one in which all banks go bankrupt conditional on the shock occurring. Following the reasoning above, all banks have a liquidity need of z (expect b_0, which needs $\theta + z$, and b_1, which needs $\frac{3z}{2}$). The condition for all banks choosing B boils down to

(*) $\frac{(1-\pi)R}{p} \geq 1$

or

(*) $l(p) < \frac{\pi p}{p-(1-\pi)R} z$

Lemma 2 summarizes all results of this Section.

Lemma 2. For a given asset price p, the equilibrium is given by

(I) If $\frac{(1-\pi)R}{p} \geq 1$, all banks choose B and go bankrupt conditional on the shock.

(II) If $\frac{(1-\pi)R}{p} < 1$, the equilibrium depends on $l(p)$.

- (*i*) If $l(p) < \frac{\pi p}{p-(1-\pi)R} \frac{z}{2}$, all banks choose B and go bankrupt conditional on the shock.
- (*ii*) If $\frac{\pi p}{p-(1-\pi)R} \frac{z}{2} \leq l(p) < \frac{\pi p}{p-(1-\pi)R} z$, two equilibria are possible.
  - (*) All banks choose B and go bankrupt conditional on the shock.
  - (*) b_2 and b_{n-2} choose S, all others choose B. Conditional on the shock, only b_0, b_1 and b_{n-1} go bankrupt.
- (*iii*) If $\frac{\pi p}{p-(1-\pi)R} z \leq l(p) < \frac{\pi p}{p-(1-\pi)R} \theta$, b_0 chooses B, b_1 and b_{n-1} choose S, and all others choose B. Conditional on the shock, only b_0 goes bankrupt.
- (*iv*) If $l(p) \geq \frac{\pi p}{p-(1-\pi)R} \theta$, b_0 chooses S, all others choose B. All banks are always solvent.

3.3 General equilibrium

Lemma 2 shows that in every equilibrium, at most two banks sell. This is a direct consequence of the absence of partial repayments: either the bankruptcy cascade is stopped by one bank and its symmetric counterpart paying back their debts in full, or it is not stopped at all. The maximal asset supply by banks is therefore $2(1-y)$ and the minimal asset demand $\frac{(n-2)y}{p}$. Assumption (2) implies

$$\forall p \in [p_{scrap}, 1], \; \frac{(n-2)\,y}{p} > 2(1-y)$$

Asset demand always absorbs the entire supply of banks, so the equilibrium asset price is $p = 1$ no matter which case of Lemma 2 applies. Replacing $p = l(p) = 1$ in Lemma 2 therefore gives the general equilibrium of the model, which can be interpreted along the same lines as above.

When the lowest possible return to assets exceeds the highest possible return to cash (Case I in Lemma 2), all banks gamble, so that, if the local shock hits, the entire network goes bankrupt. This may be interpreted as systemic risk increasing in good times: when asset returns are high and the probability of a local shock low, banks are more inclined to make portfolio choices that leave them exposed in case the shock hits. While this behavior is privately rational, the outcome for a large network is clearly suboptimal. All amplification could have been avoided and the return on all assets could have been collected by just carrying over θ units of cash (an arbitrary small amount with respect to the initial aggregate liquidity) to the intermediate period.

With a lower asset return or a higher likelihood of a shock, results depend on the size of the shock and of interbank debts, shaping banks' liquidity needs. Assume first the shock is large. Then, with high interbank debts, the amount of cash that can be rescued after a shock is low, and all banks gamble (Case II.i). At an intermediate level of interbank debt, there are multiple equilibria (Case II.ii). b_0 and its creditors gamble because of their higher liquidity needs. Their neighbors may shelter the remainder of the network by choosing S or put it in (conditional) default by choosing B. Equilibrium is a self-fulfilling prophecy: when choosing S, for instance, the neighbors make the network safer and thereby decrease their own liquidity need, ex post rationalizing their action. With an even lower level of interbank debt, indeterminacy breaks (Case II.iii). The creditors of b_0 unambiguously prefer to choose S and shelter the rest of the network. Finally, when the shock is small, b_0 itself chooses the safe action and shelters the rest of the network (Case II.iv).

Results do not change when I introduce network uncertainty of the type considered by Caballero and Simsek. Assume, as they do, that banks are rotated across the network and only know whether their neighbors or themselves could be hit by the local shock. This would affect decisions for banks that know that they are at least two slots away from b_0, but are uncertain about their exact distance. However, Lemma 2 shows that when the network is common knowledge, these banks always take the same decision, irrespective of their exact distance to b_0. Therefore, this form of network uncertainty does not affect them[7]. The model instead highlights a different amplification mechanism. Risk-taking incentives and network externalities explain that banks may arrive to crisis periods with illiquid portfolios[8], vulnerable to a local shock and its spillovers.

With a wider definition of network uncertainty, results can change somewhat. Assume, for example, that banks are completely unsure about which bank may get hit by the shock. Then, at $t = 0$, all banks have the same expected payoff (a weighted average of payoffs in the cases where the shock hits them, their immediate neighbors or a bank further away in the network). Therefore, they must all take the same decision. When gambling is more profitable than selling assets, all banks gamble and go bankrupt when the shock occurs. In the reverse case, all banks sell and there is a fire sale. Note that the network structure does not matter in this case. I therefore prefer using the original Caballero and Simsek definition of network uncertainty, which gives a role to distance.

3.4 Inefficiencies and government intervention

An equilibrium with generalized bankruptcy is Pareto-inefficient because banks do not consider the externality they impose on their creditors when choosing B. A government can improve every bank's situation by creating a bailout fund, financed by a tax on all

[7]The only exception is Case II.ii, where equilibrium depends on the actions of b_2 and b_{n-2}. However, as their decision is indeterminate, it is impossible to say how it will be affected.

[8]As argued before, illiquidity can be interpreted as a proxy for a fire sale in a crisis episode. One may wonder whether this fire sale by assumption is robust to introducing an asset market in the intermediate period. Intuitively, the asset market may create an incentive to hold cash in order to buy assets at fire sale prices and thereby cancel the fire sale in equilibrium. However, in a network, the situation is not as clear-cut. In order to benefit from low prices, banks must stay solvent, but when a bank close to b_0 stays solvent, it makes the rest of the network safe and eliminates the fire sale from which it wanted to benefit. On the other hand, for banks far away from b_0, this correlation between solvency and absence of fire sales may not exist. Whether fire sales arise in equilibrium depends on the relative mass of these two groups and thus on the trade-off between benefiting from low prices and having to stay solvent to do so. I have not explored this further.

banks in the initial period and used to bail out b_0 after the shock.

As this intervention makes all banks solvent, b_0 has a liquidity need of θ and every bank must contribute $\frac{\theta}{n}$ to the fund. When the shock does not hit, contributions are refunded. Nevertheless, there is an efficiency loss, because banks are not able to invest all their resources in the asset. In expectation, this loss is $(1-\pi)(R-1)\frac{\theta}{n}$. When the shock hits, b_0 is bailed out so that all banks remain solvent instead of going bankrupt. This gives them an expected gain of $\pi R\left(1-\frac{\theta}{n}\right)$. Clearly, for a large network, the bailout increases the expected equity value for every bank[9].

The bailout fund could in principle also be set up by the banks themselves. However, there would be two equilibria, one in which all banks participate and one in which no bank participates. By making contributions mandatory, the government creates a coordination mechanism towards the superior equilibrium.

Equilibria without generalized bankruptcy cannot be Pareto-improved upon. Safe banks achieve the highest possible equity value and the constant returns to scale to assets do not allow other banks to compensate them for giving this up.

4 Conclusion

Limited liability and uncertainty about the future create a risk-taking incentive in a financial network. Banks choose to hold illiquid portfolios which amplify local shocks, because their payoff structure incentivizes them to do so and because they neglect network externalities. A bailout fund with mandatory contributions can therefore create in many cases a Pareto-improvement.

By considering an exogenously given circular network, I have abstracted from more general policy concerns, such as determining the optimal network structure. A richer model, taking into account how banks form links when they anticipate risk-taking by their counterparties, could help to shed further light on these issues.

[9] When the government does not care about the outside agent, it may be able to do better by bailing out b_1 and b_{n-1} instead of b_0. Assumption (1) implies their liquidity need is smaller, so contributors to the bailout gain more. b_0 would however be worse off by joining the fund and should be exempted.

When the government can access cash at a low cost outside of the banking system, it may also be more efficient to lend θ to b_0 at $t=0$ for reimbursement at $t=2$. When $R>\theta$, this loan improves b_0's situation relative to non-intervention, while all other banks get their highest possible equity value.

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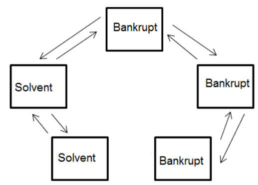
Appendix

A.1 Potential equilibrium outcomes

Suppose there is an equilibrium in which the outcome for b_k and b_{n-k} $(k \geq 1)$ is different. Consider first $k = 1$ and suppose without loss of generality b_1 is bankrupt and b_{n-1} is solvent. In this case, b_0 must be bankrupt. If it were not, it would pay back its debts and all other banks would be solvent. It must also be that b_1 and b_{n-1} have different liquidity needs: otherwise, they would choose the same action and get the same outcome. As they receive the same from b_0, they must receive different amounts from their other neighbors. Moreover, b_1 must receive strictly less than b_{n-1} (otherwise, the former would go bankrupt with a lower liquidity need, which is impossible). As banks pay back either everything or nothing, this implies b_2 is bankrupt and b_{n-2} is solvent. The situation of these “first” five banks in the network is illustrated in Figure 4.

Figure 4: Illustration: Asymmetric outcome for $k = 1$



This configuration implies that, for every combination of outcomes for the remaining banks[10], there will be a chain “Solvent, Solvent, Bankrupt, Bankrupt”, where none of the involved banks is b_0. The second and the third bank of this chain both have a solvent and a bankrupt neighbor which behave identically[11] and therefore the same liquidity need. Nevertheless, one of them goes bankrupt and the other does not: this is a contradiction.

Now, consider the case $k = 2$ and assume b_2 is bankrupt while b_{n-2} is solvent. For the same reasons as before, this implies b_0 is bankrupt. Moreover, b_1 and b_{n-1} must also be bankrupt: as I have shown above, these two banks must have the same outcomes, and if they were both solvent, the rest of the network would be solvent as well. I can now repeat the preceding proof. b_2 and b_{n-2} must have different liquidity needs, implying b_3 is bankrupt and b_{n-3} solvent. Again, a chain “Solvent, Solvent, Bankrupt, Bankrupt” appears, and this is a contradiction.

The proof immediately generalizes to an arbitrary k. It also shows b_k and b_{n-k} can only be bankrupt if b_0, b_1, ... b_{k-1}, b_{n-k+1}, ..., b_{n-1} are bankrupt as well.

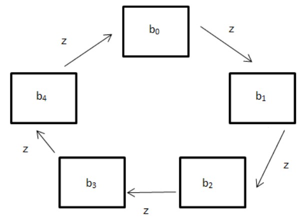
A.2 Robustness of the Caballero and Simsek result in the new network

In their model, Caballero and Simsek consider a network where every bank owes only to its forward neighbor and has a claim on its backward neighbor, as illustrated in Figure 5.

[10] Taking into account that there can never be a bankrupt bank between two solvent banks, because payments are perfectly enforced and claims cancel out.

[11] This is why b_0 must be excluded: a payment to this bank goes entirely to the outside agent, while a payment to another bank can be at least partly recovered.

Figure 5: The network in Caballero and Simsek



I have changed the structure of the network somewhat to relax an undesirable assumption about network uncertainty. To see this, it is necessary to go a bit deeper into the model's mechanisms. In the Caballero and Simsek model (with the network of Figure 5 and $\pi = 1$), a bank chooses S if and only if it has a positive liquidity need[12]. b_0 always needs liquidity and thus always chooses S. When the shock is larger than the cash it gets, b_0 goes bankrupt and cannot fully repay b_1. b_1 then chooses S as well, etc. Partial repayments are increasing along the cascade: b_0 may not pay back anything at all, but b_1 already pays at least the cash it got from selling its assets, b_2 pays the cash it got from its own asset sales and the amount it received from b_1, etc. Under certain parameter assumptions, the cascade eventually stops. Unaffected banks choose B and create the asset demand necessary to maintain the high price $p = 1$.

With network uncertainty, banks get rotated across the network and only observe the identity of their forward neighbor[13]. They are assumed to be uncertainty-averse and therefore maximize the equity value they get in a worst-case scenario, where b_0 is rotated into the position closest to them. Therefore, all banks (except b_0 and its new backward neighbor) now act as if they where in the position of b_2 in the common knowledge network. When a shock is so large that it must spill over to b_2, all banks think they may be affected and sell. The asset price falls to p_{scrap} and the bankruptcy cascade is amplified.

It may seem desirable to let banks observe their backward neighbor (the bank they owe money to) as well. However, in that case, the fire sale equilibrium does not exist anymore. The bank rotated into the slot just before b_0 now learns its position. It also knows the bankruptcy cascade is never long enough to reach it (because of the parameter assumptions mentioned above). In sum, it learns it is in a safe position and will not need liquidity at $t = 1$. It therefore does not choose S, but B. When market transactions are observable, it becomes common knowledge this bank is safe. This is enough information for the bank placed just before it to figure out its network position as well and change its action. Proceeding by induction, it is easy to see all safe

[12] This means that even banks that go bankrupt no matter what action they take, and are therefore indifferent between B and S always choose S.

[13] As noted by Caballero and Simsek (footnote 11), this is equivalent to banks keeping their positions, but being uncertain about where the shock hits.

banks eventually learn they are safe and choose B, so there is no fire sale.

The network considered in the main text removes this undesirable feature by getting rid of the asymmetry between network distance and distance in terms of the bankruptcy cascade (b_{n-1}, for example, is close to b_0 in the first measure and far from it in the second one). In this network, even when all banks observe their backward neighbor, safe banks do not know they are safe.

This network change however does not affect Caballero and Simsek's results qualitatively when $\pi = 1$. I now show this formally.

Consider the model given by Figure 2, $\pi = 1$ and replace (2) by

$$ny \geq 4 + \lceil 2z \rceil \tag{5}$$

(5) ensures aggregate liquidity is large enough to buy up all sold assets. It is a little more stringent than its equivalent in the main text (because there will be typically more than two sellers), but always holds for sufficiently large networks. I now prove Lemma 3, the equivalent of Proposition 1 in Caballero and Simsek.

Lemma 3. When all banks know the network, the equilibrium for a given asset price p is

(*i*) If $l(p) \geq \theta$, then b_0 chooses S. All banks remain solvent.

(*ii*) If $z \leq l(p) < \theta$, then b_0 , b_1 and b_{n-1} choose S. b_0 goes bankrupt, all others remain solvent.

(*iii*) For $k \geq 1$, if $\frac{2z}{(k+1)(k+2)} \leq l(p) < \frac{2z}{k(k+1)}$, then b_0, b_1, ... b_{k+1} and b_{n-1}, ..., b_{n-k-1} choose S. b_0, b_1, ... b_k and b_{n-1}, ..., b_{n-k} go bankrupt, all others remain solvent.

The proof follows the same case distinctions as in Section 3.2.

Case 1. b_0 does not go bankrupt.

b_0 has a liquidity need of θ and chooses S. Case 1 can only apply when sales are sufficient to cover the liquidity need, that is, $l(p) \geq \theta$.

Case 2. b_0 goes bankrupt, but b_1 does not[14].

b_1 pays its entire short-term debt to b_0. The latter therefore still has a positive liquidity need θ and chooses S. However, b_0 goes bankrupt, so its asset sales cannot be sufficient to cover its liquidity needs: it must be that $l(p) < \theta$, showing Cases 1 and 2 are mutually exclusive. Assumption (1) implies b_0 cannot pay anything to b_1. The latter then has a liquidity need of z, chooses S and remains solvent only when $l(p) \geq z$. In sum, this case can only occur when $z \leq l(p) < \theta$.

Case 3. b_0 and b_1 go bankrupt, but b_2 does not.

[14] As in the main text, I only consider one side of the symmetric network. Every statement on bank b_k ($k \geq 1$) applies as well to bank b_{n-k}.

b_1 receives nothing from b_0 and z from b_2. It therefore has a liquidity need of z, which it is unable to cover, so $l(p) < z$. This condition implies b_0 is bankrupt as well (as its liquidity need is now even higher than θ, and $\theta > z$) and that Cases 2 and 3 are mutually incompatible. As b_1 has a positive liquidity need, it will sell its assets. It therefore has total resources of $l(p) + z$ (the proceeds from the asset sale plus the loan repayment from the solvent bank b_2), which it splits equally among its creditors after bankruptcy.

Accordingly, b_2 has a liquidity need of $2z - z - \left(\frac{z+l(p)}{2}\right) = \frac{z-l(p)}{2}$. For Case 3 to apply, it must be able to cover it, that is $l(p) \geq \frac{z-l(p)}{2}$. In the end, Case 3 can thus only occur when $\frac{z}{3} \leq l(p) < z$.

Using the same reasoning as in Case 3, I can pass on to the general case, in which banks b_0, b_1, ... b_k (for $k \geq 1$) go bankrupt, but bank b_{k+1} does not. I denote by r_k the repayment made by b_k. Repayments are linked by the following system of equations:

$$\begin{cases} r_1 = \frac{r_2+l(p)}{2} \\ r_2 = \frac{r_1+r_3+l(p)}{2} \\ \dots \\ r_{k-1} = \frac{r_{k-2}+r_k+l(p)}{2} \\ r_k = \frac{r_{k-1}+z+l(p)}{2} \end{cases}$$

This takes into account that all these banks choose S, b_0 never repays anything and b_{k+1} is solvent and repays the full amount. In matrix form, the system writes

$$\begin{bmatrix} 1 & -\frac{1}{2} & 0 & 0 & 0 & . & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & . & 0 & 0 & 0 \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & . & 0 & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & 0 & . & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & . & 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ . \\ r_{k-1} \\ r_k \end{bmatrix} = \frac{l(p)}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ . \\ 1 \\ 1 \end{bmatrix} + \frac{z}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ . \\ 0 \\ 1 \end{bmatrix}$$

Calculating $L_1 + 2L_2 + 3L_3 + ... + kL_k$ (where L_k stands for the k^{th} line of the system), I find $r_k = \frac{k}{k+1}z + \frac{kl(p)}{2}$.

It is easily shown r_k is the largest repayment, so that banks from b_0 to b_k will be bankrupt if $r_k < z$, which is the case if $l(p) < \frac{2z}{k(k+1)}$. The liquidity need of b_{k+1} is $2z - z - r_k = \frac{z}{k+1} - \frac{kl(p)}{2}$, and it can be covered by asset sales if $l(p) \geq \frac{2z}{(k+1)(k+2)}$. These are the two inequalities of the Lemma. Note that the conditions for all cases to apply are mutually exclusive, so that there is a unique equilibrium for every parameter configuration.□

From these results, it follows that there are always at most $\left\lceil \frac{2z}{l(p)} \right\rceil$+4 banks selling[15]. Assumption

[15] This is by no means the smallest upper bound. For a tighter one, I could make a less restrictive

(5) ensures that there are always some banks that are not affected by the bankruptcy cascade, so that the demand for assets exceeds the supply of selling banks at every price[16]. Therefore, the equilibrium price under common knowledge of the network is $p = 1$.

Under common knowledge of the network, the equilibrium has exactly the same features as the one in Caballero and Simsek. The same is true when introducing network uncertainty. Assume banks are rotated across the network and can only observe their new neighbors. Then, all banks at a distance greater than 2 of b_0 will act as if they were actually just two slots away from that bank, and sell assets when they anticipate a bankruptcy cascade will reach this position. A large enough shock triggers a fire sale, and network uncertainty is an amplifier of local shocks.

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assumption to replace (5). However, as I argued before, (5) is not really restrictive in the first place, as I can choose an arbitrarily large n.

[16] Asset supply is smaller than $\left(\left\lceil \frac{2z}{l(p)} \right\rceil + 4\right)(1-y)$ and asset demand larger than $\frac{\left(n - \left\lceil \frac{2z}{l(p)} \right\rceil - 4\right) y}{p}$. Supply exceeds demand if $ny > \left(\left\lceil \frac{2z}{l(p)} \right\rceil + 4\right) l(p)$, which is ensured by (5) and the fact that $l(p) \leq 1$.